Fluid Mechanics
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Lecture - 23
Flow in Noncircular Conduits and Multiple Path Pipeflow

Welcome all of you to this fluid mechanics lectures as I am to say is this is the last lectures for this series of the lectures for these 8 weeks and 20 hours lecture series on the fluid mechanics. So let us have a talking about the books. Again I am to repeat you that let us follow the some of the books okay which is highlighted here.

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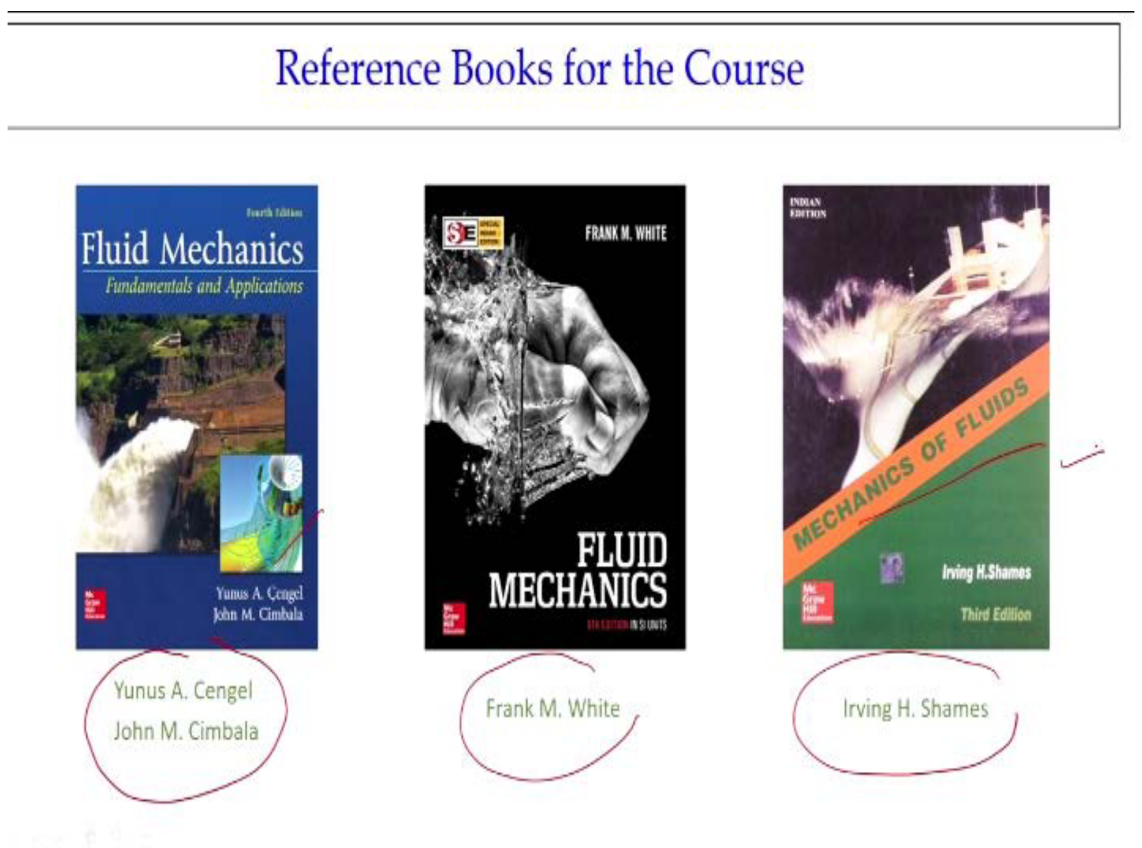

Mostly again I am to repeat it, if you want to have the basic concept and understanding the fluid mechanics as a illustrations wise the fluid flow characteristics, please read the books in written by Cengel Cimbala, Fluid Mechanics Fundamentals and Applications, F. M. White books and I. H. Shames books, Mechanics of the Fluids. Please go through these books which are really quite interesting way written the fluid mechanics from introductory level to the advance level.

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Contents of Lecture

1. Nikuradse's flow Experiment and Flow Vegetation Experiment
2. Noncircular Conduits
3. Velocity Profiles and Shear Stress at the Boundary
4. Multiple-Path Pipe Flow
5. Example Problems Solving on Losses in Pipe Fittings
6. Summary

Now let us come back to today's contents what I am going to deliver to you. Very interesting experiments was conducted almost 70 years back. That is what we will be discuss here. And the new experiment what we are conducting at IIT Guwahati is as a glimpse I will show to you. Then we will talk about three things noncircular conduits, how we can apply the same equations for noncircular conduits.

And we also will talk about how does velocity vary in a pipe flow and how to compute wall shear stress at the boundary. And also we will talk about how to solve the multi-path pipe flows. And then we will solve some of the GATE questions on the fluid flow through pipes and we will have the summary.

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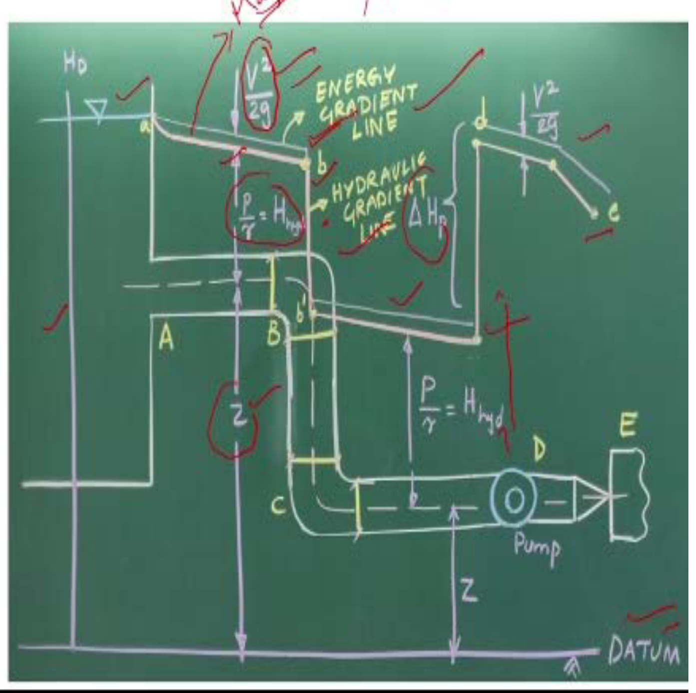
Recap of the Previous Lecture

1. Minor losses in Pipe system
2. Energy and Hydraulic Grade lines



Now let us coming to the previous lectures, the recap of the previous lectures, as soon in these figures. Whenever you have any pipe flow components, we like a pumping systems, the reservoirs, please draw energy gradient line and the hydraulic gradient line. Those two lines will indicates us where the energy loss is happening it and where the energy gaining is happening.

For example, if you talk about the pump, because of these pump resistance we have a getting a extra energy to the fluid flow through the pipe systems. So that is the reasons again I am to repeat it to tell it first you draw a data and you have the reservoirs and from that reservoir you start drawing the energy gradient line, also the hydraulic gradient line.

$$H_{hyd} = \frac{P}{\gamma}$$

The hydraulic gradient line energy gradient line the difference will be the velocity head. That is what the $\frac{V^2}{2g}$, the velocity head. So any datum I can have a z is potential head then I have the pressure head, then I have the velocity head. So these three components, I can draw the lines like this. The for example, if you look at these diagrams from the reservoirs you do not have any velocity components.

That is the reasons you have a hydraulic gradient and energy gradients, both coincide at that locations. As you go, there is an energy losses and through this energy losses what it happens in pipe, that is what we call major losses and the bends entry point and exit point, we call minor losses, minor losses. And these losses already we quantified in the last class.

And as you know it, this the hydraulic gradient lines, drop of this hydraulic gradient line or the energy granted line because of the dropping of the potential energy from this pipe orientations. Again if you look it there is a major losses, this pumping systems is there which is giving extra energy or energy head to these. That is the reasons again you have velocity head and this.

So whenever you have a pipe flow problems try to sketch approximately energy gradient line and the velocity hydraulic gradient line. As you draw the energy gradient line and the hydraulic gradient line you can try to understand how the flow taking place, where the energy losses are happening it, whether the major losses, the minor losses or adding the energy to the system, all you should identified it when you drawing this energy gradient line, hydraulic gradient line.

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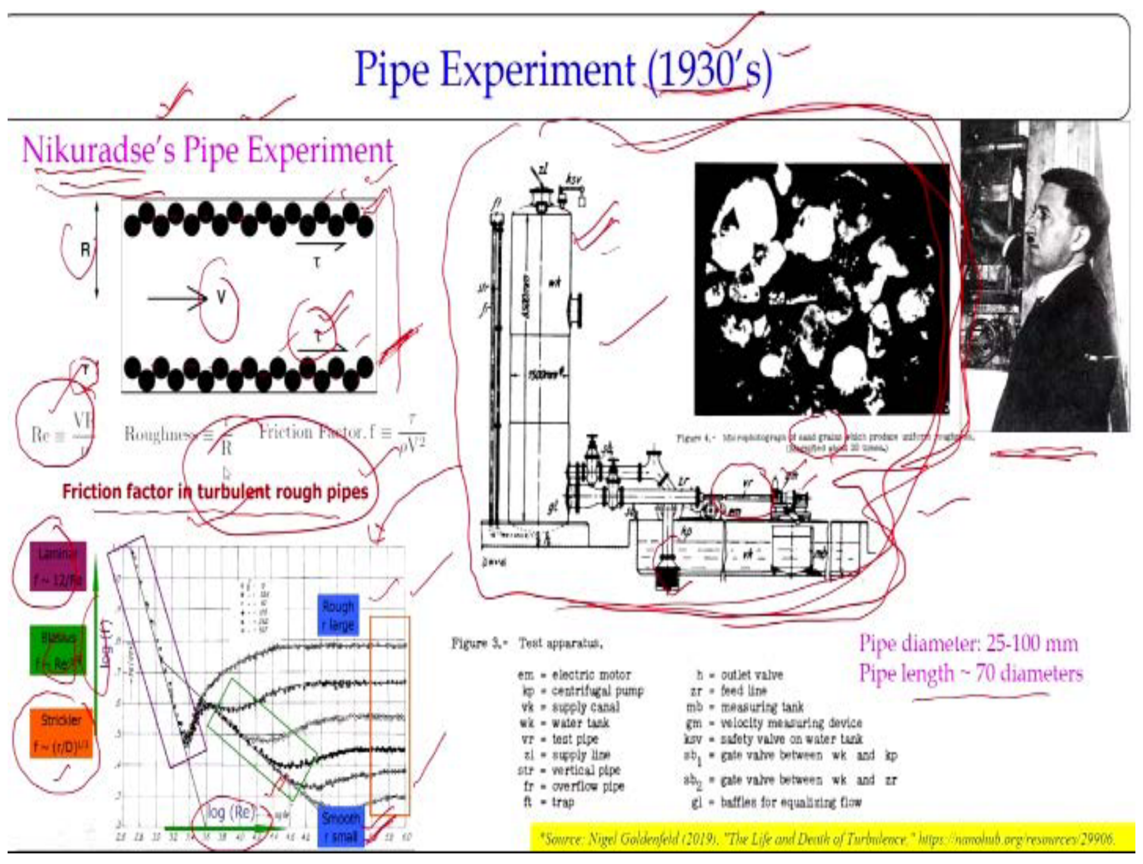

Now let us come back to very interesting experiments what it happened in 1930s, okay much before the World War II okay. The Germans the professor used to do a simple experiments okay which one of classical experiment conducted in a pipe flow and which helped to make a this complex flow to a simpler empirical based energy loss quantification, velocity distributions quantification as well what could be this wall shear stress.

If you look at this any of the pipe flow we can have a laminar flow, the turbulent flow or the transitional in between the laminar and the turbulent flow. What the experiment is conducted by the Nikuradse is that with having very simple concept that the pipe put it with a roughness, this equivalent roughness is from the sand grains. This figure is shown it with a zoomed with a 20 times.

That means you can see there is quite random the sand grains are roughened it or put it in here such a way that it can create a roughness on the wall of the pipe. If you have a that roughness which representing is R, the roughness site representing as R and R is the radius of the pipe flow. So you can know it the because of the roughness, your flow characteristic you can change it.

The also we know very well the Reynolds numbers also control the flow behaviors. So that way to know it what is will be the wall stress the tau value, what could be the velocity distributions, what could be the energy losses when you have a pipe length of L with the informations about the roughness, informations about flow Reynolds numbers.

Those that was the basic question was there before starting the experiment by Nikuradse experiment setup, which is very basic experiment setup, you can see it this from this. Very basic experiment set up having the big water tank connected with a pump and having a test sections here, which measure the velocities and all. And if you look at the dimensions of these water tank, which is 6.5 meter tall 1.5 meters diameters.

So you can look it that how precisely experiment we are conducted to quantify what could be the energy losses and what could be the velocity distributions and what could be the relationship of velocity distributions with wall shear stress. As you can understand from this experiment what was conducted in 1930s, it is quite remarkable way we got it with relationships for the turbulent rough pipes.

That you have a friction factors and you have the flow Reynolds numbers which gives us a very three reasons. One is if you have a R is too large and the Reynolds number is too large you can have a friction factor is a function of R by V. That means roughness factors, R by V is roughness factor and when you have flow is laminar, it is a functions of the Reynolds numbers.

And if you have a smooth pipe where the roughness is very, very small, roughness is the factor is much slow, very small, then you can have a the smooth R values. The R is a very small then you can follow this Blasius equations which is a functions of the Reynolds numbers.

So now if you look it that by conducting a series of experiments, by Nikuradse finally bring a the friction factors relations with the Reynolds numbers which is Moody chart and we have been using that for designing all this pipe flow networks for the industry, the water supplier, sewage treatment plant all where we are using the same equations, what was developed way back in 1930s conducting a series of experiment for turbulent rough pipes. This is very complex till now to do in a shape simulations.

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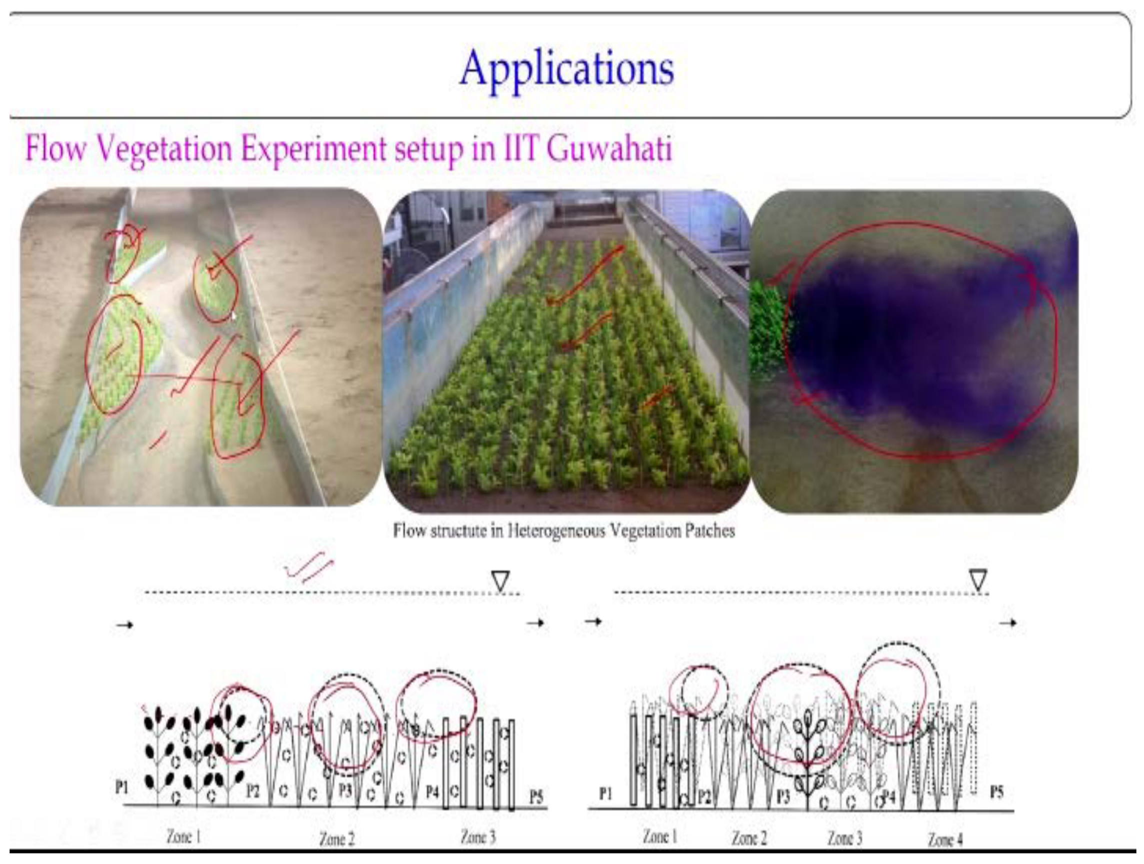

Now if you come back to that what we have been doing at IIT Guwahati, we also do similar sort of creating the roughness in open channel flow. That is what you can see this vegetations are there. Some degree of we are creating the roughness and we try to measure the velocity distribution. Try to quantify the wall shear stress what could be there, bed shear stress could be there.

All we are quantifying, measuring the velocity distributions, measuring the water flow and what we have? We have introduced the roughness and you can see this roughness behaviors and how the flow dispersions are happening it with a putting the color dyes. I am not going much detail on that.

But if you look it for this case, if the flow is submerged conditions, and you have a the roughness like this, there are the vortex the eddies formations and the strength of eddies formations you can get it and that is what we got it conducting a series of experiments in this flow as well this flow.

So what I am to tell it that the experiments gives us a lot of confidence of a very complex problems like a turbulent flow in a rough pipes which is very difficult to model as of now also in the softwares, but we can conduct a series of experiment and we can establish the empirical relationship and those empirical relationship is used in generally in industry to design the pipe networks.

Now coming to the other case, many of the times also we do not go for only the circular conduits or the circular pipes from one point to other points okay.

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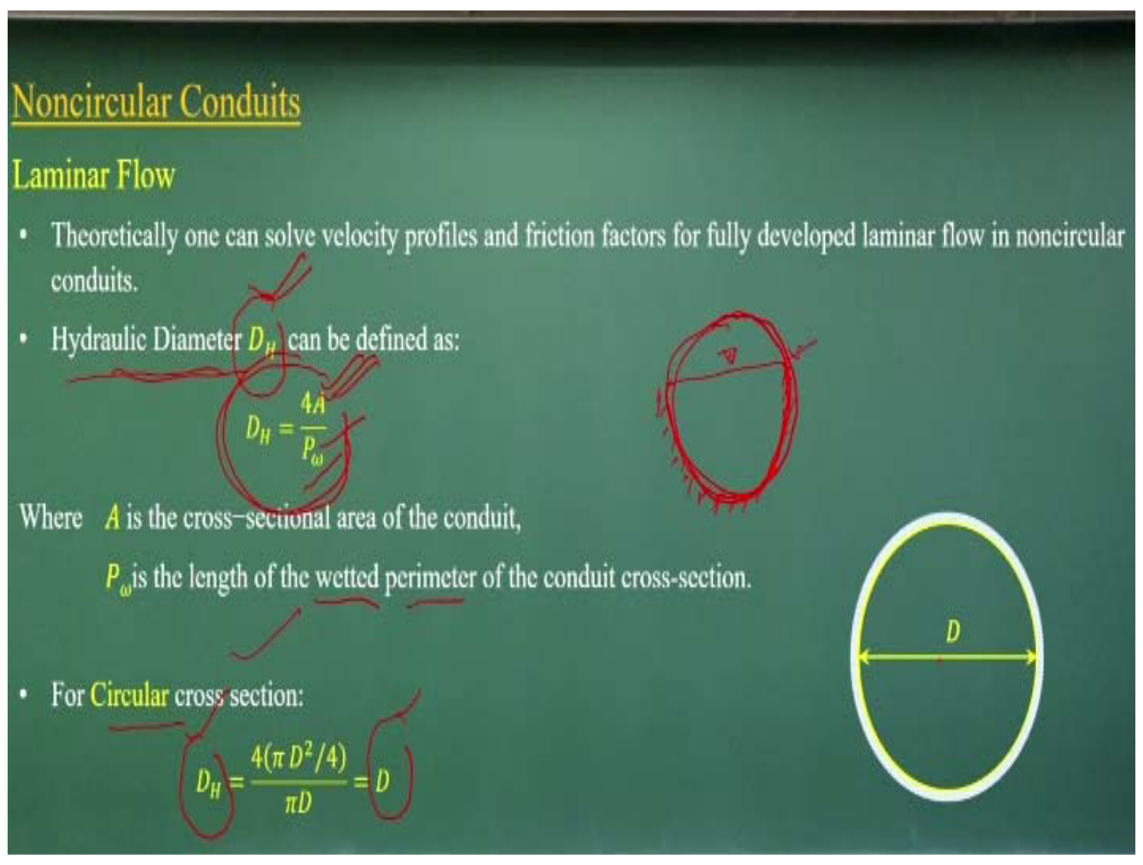

But when you go for a noncircular case you need to define as a equivalent flow okay. So that means we introduce a hydraulic diameters okay, it is called as hydraulic diameters,

$$D_H = \frac{4A}{P_\omega}$$

which is a functions of area and wetted perimeter. The part of the perimeter which is under wetted conditions, wetted perimeters. Like for example, if I have the flow pipes, it is going through whole, so that way the total perimeter I can consider as wetted perimeter.

But if I have the flow half filled the pipe flow is going it, then I will consider the wetted part of this ones only. I will consider the wetted perimeter. I am just highlighting that. We need to consider wetted perimeters the area, then we can compute the hydraulic diameters, okay as equivalent diameter we can get it. That is what is for the circular sections.

$$D_H = \frac{4(\pi D^2/4)}{\pi D} = D$$

If we just substitute if the D is a diameters and the perimeter for the whole flow of these pipe systems, you will get it hydraulic diameter in a circular pipe when it contains full of water, okay full of liquid what is flowing through that. Then you can get it the hydraulic diameter as equal to the geometric diameters. So that what is equal components which comes for that. Please do not be confused. The diameter, hydraulic diameter as a equivalent diameter representing for other noncircular conduits.

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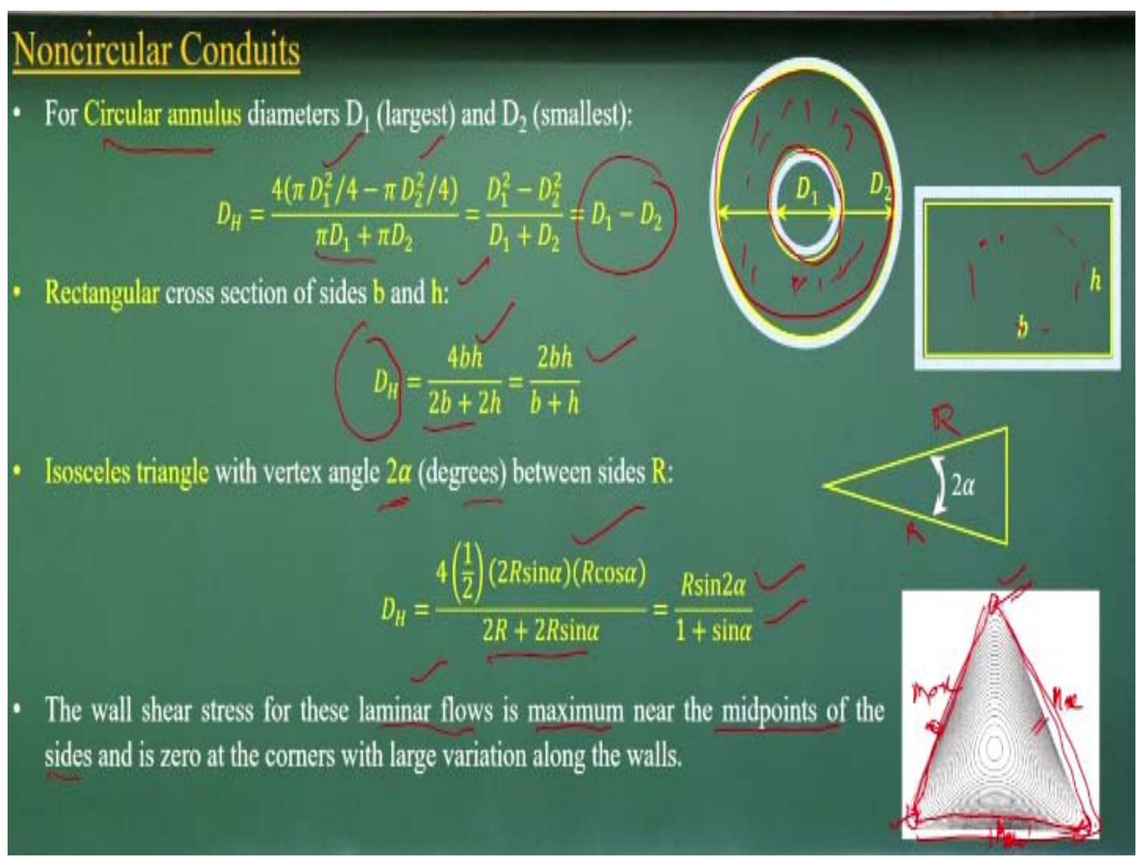

Now come back to the noncircular conduit like you may have a conditions where you have a only flow through this ones okay. That is called circular annulus diameter.

$$D_H = \frac{4(\pi D_1^2/4 - \pi D_2^2/4)}{\pi D_1 + \pi D_2} = \frac{D_1^2 - D_2^2}{D_1 + D_2} = D_1 - D_2$$

You have a D_1, D_2 and you have a flow through these ones. So you can easily find out what will be the flow area and what will be the wetted parameters for both the cases. That is what will give you this one.

Similar way if you have a rectangular cross-sections and flow through these systems, I can define it what will be the area and the wetted perimeters. That is what will give me the hydraulics diameter. If you have a isosceles triangles with a two alpha degrees between radius R, also I can put it numerically what could be the area what could be the perimeters and I can get it for these ones.

Rectangular cross section of sides b and h

$$D_H = \frac{4bh}{2b + 2h} = \frac{2bh}{b + h}$$

Now let us come it with that when you have a noncircular pipe like if you have a pipe like a triangular shape okay. So in this case what will happen the if you have a laminar flow you will have a wall stress will be maximum near the mid points of the sides. It will have a maximum at the midpoint of the side, wall shear stress. Becomes will be zero at this point, zero at this point, okay.

Isosceles triangle with vertex angle 2α (degrees) between sides R

$$D_H = \frac{4\left(\frac{1}{2}\right)(2R\sin\alpha)(R\cos\alpha)}{2R + 2R\sin\alpha} = \frac{R\sin 2\alpha}{1 + \sin\alpha}$$

So more details I am not going it because it is quite undergraduate level and we did not discuss much detail of the flow patterns okay what supposed to be there when you have a the complex geometry like a isosceles triangles that you can see that the more maximum wall stress will develop at this point, well it will be zero at these three points.

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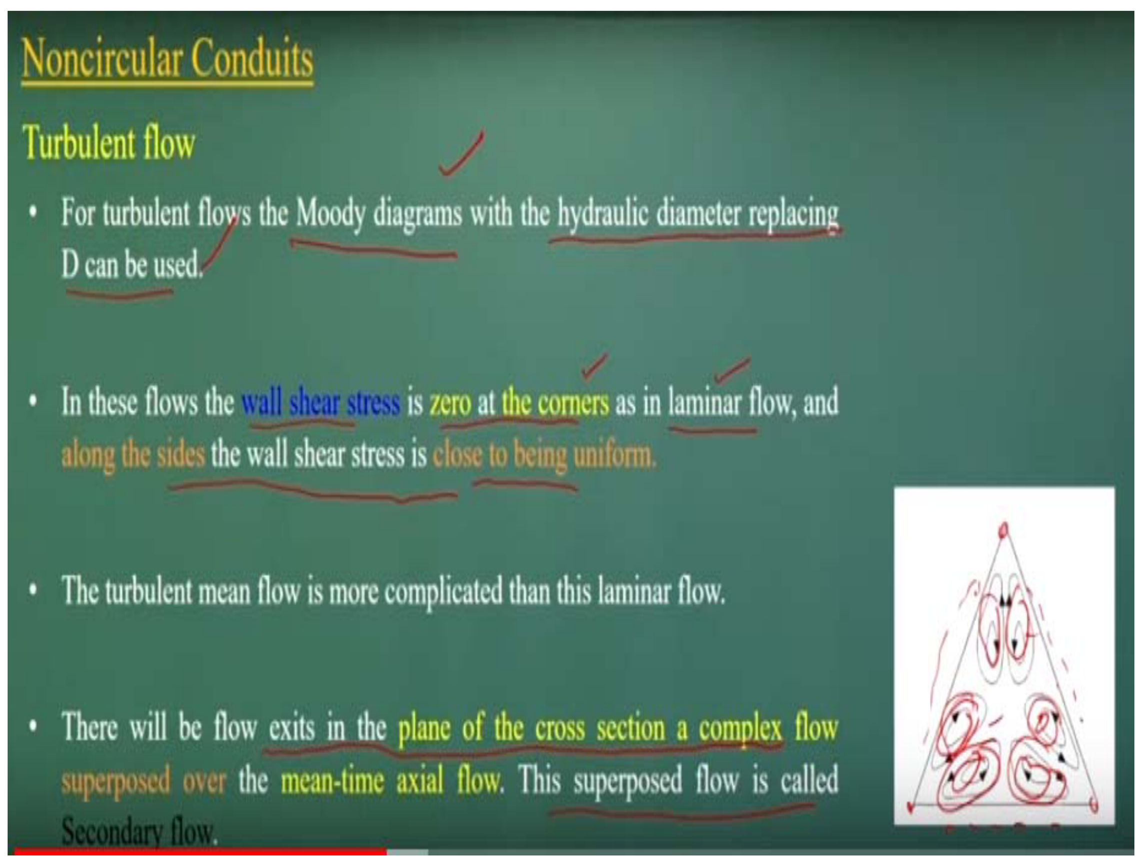

Now if you look it that if you have the turbulent flow same case you have a turbulent flow the velocity distributions as well as the wall shear stress distributions exchanges it. What we do it in any case of the turbulent flow we also use the Moody's diagrams. Hydraulic diameters replacing with a D. That is what is used to quantify what could be the energy losses.

The wall shear stress is zero at the corner as already we tell it in laminar cases. So we will have a wall shear stress at the corners is zero. But along the side the wall shear stress is close to being the uniform. This side you will have uniform, not the maximum at the midpoint as you got in case of laminar flow. So that is the flow behavior so what is observed by conducting a series of experiment quantifying the velocity distributions knowing it what could be the wall shear stress.

Now if you look at that, when you have this type of flow, always you can have this type of vortex formations, okay. This type of vortex formations you can see it and these vortex formations we call the secondary flow generations what it happens it and these are responsible for the mass actions, momentum actions through these the flow systems.

So as you go from away from this circular pipes for this case like a triangular flow zones you have a more complicated the velocity distribution shear stress distribution as compared to the circular pipes as a symmetrical problems what we try to understand it. Now coming back to the turbulent flow again we can follow the same head loss equations.

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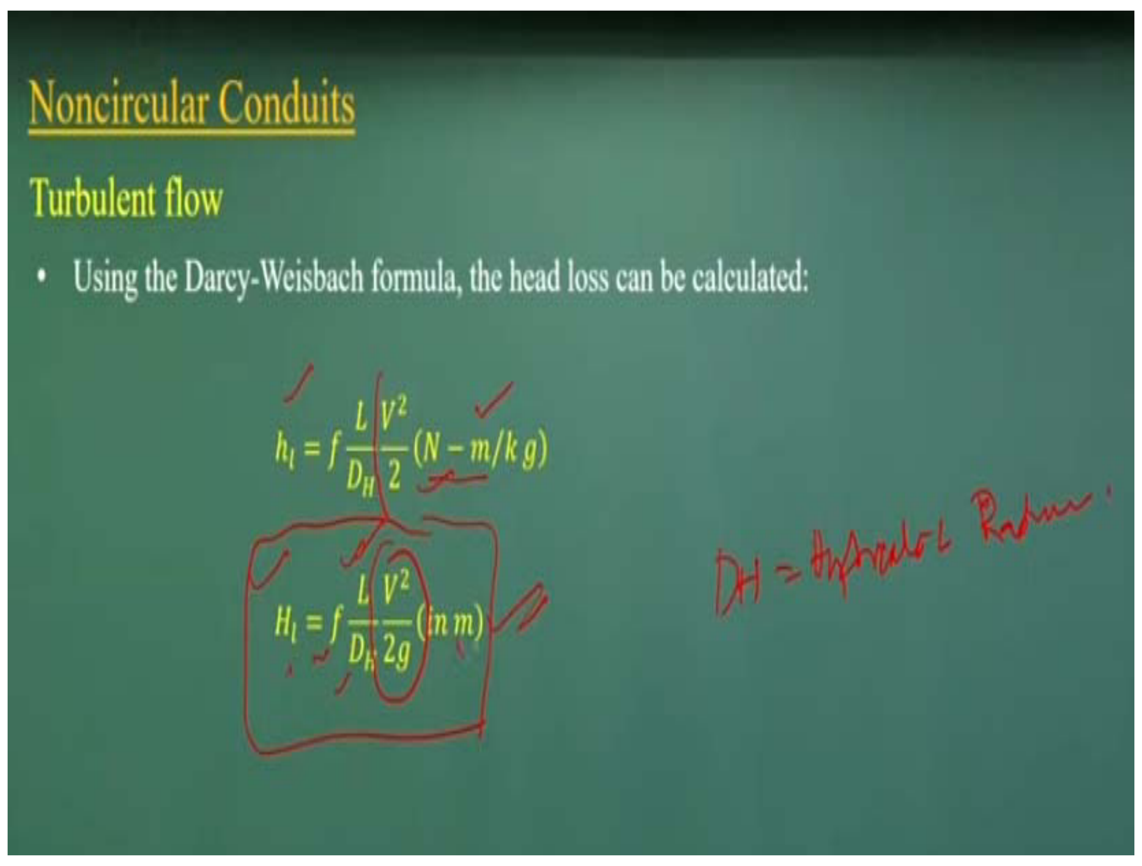

The head loss is a functions of the velocity head which is

$$h_l = f \frac{L}{D_H} \frac{V^2}{2} (N - m/k\,g)$$

$$H_l = f \frac{L}{D_H} \frac{V^2}{2g} (in\, m)$$

Most often we use these because this is what is gives the energy losses in terms of meter, but we can use these functions which gives us in terms of Newton meter per kg. So divide by the acceleration due to gravity will get in terms of meter.

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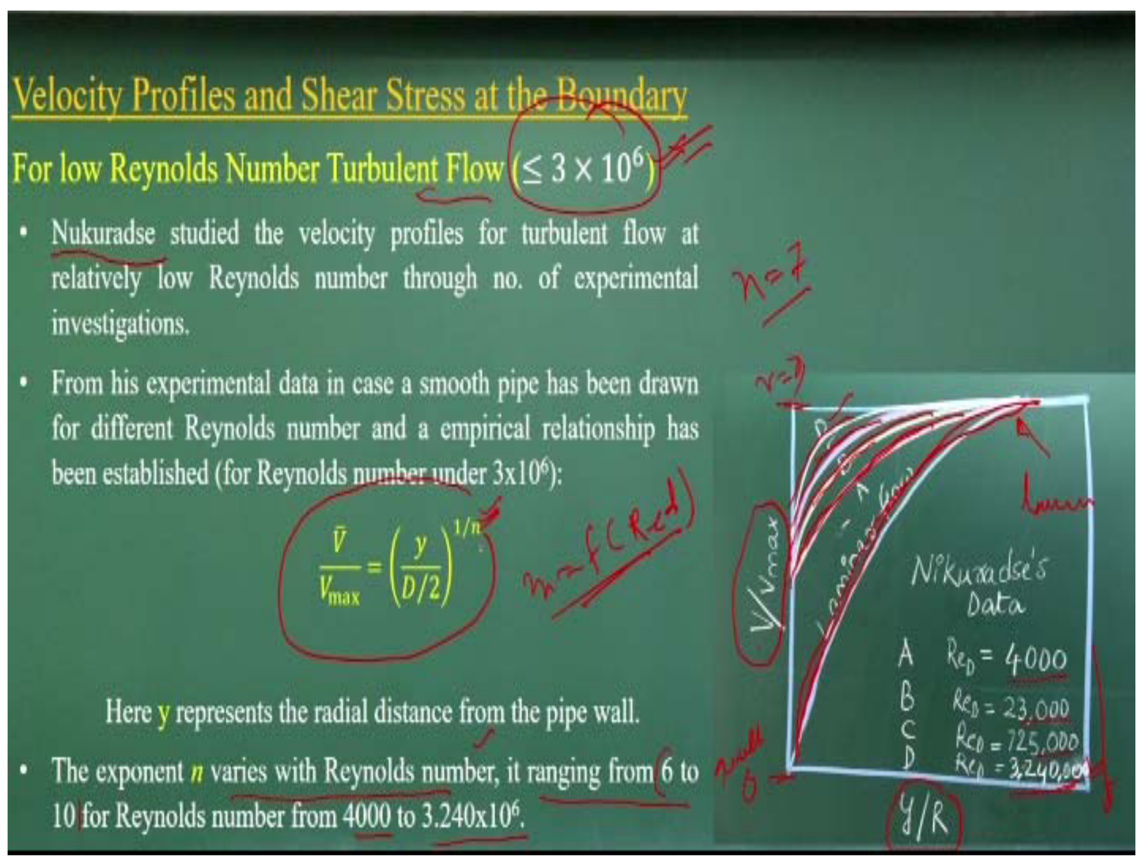

Now coming back to the velocity profile, or the shear stress at the boundary. Here we divide into two zones as suggested by Nikuradse by conducting a series of experiment, finding out velocity profiles for the turbulent flow where you have the flow Reynolds number less than three millions okay. If you look at this Reynolds numbers, okay it is too large okay.

But because as I said it earlier the experiment setups was designed in such a way that he could conduct the experiment which will have a flow Reynolds numbers in the range of three millions more than that. By conducting this experiment what he got it that the non-dimensional velocity distribution

$$\frac{\bar{V}}{V_{\max}} = \left(\frac{y}{D/2}\right)^{1/n}$$

And this is the and R equal to zero that means it is a center points.

If you plot y by R okay and $\frac{\bar{V}}{V_{\max}}$ no doubt the velocity distribution for laminar flow this follows this ones. This is the laminar flow. But as increase the Reynolds numbers, okay, as increase the Reynolds number it goes off to these you can go to up to 3.24 order of million scales. If you look it that the velocity distributions as going more flatter to flatter okay. That is the very uniqueness it comes it.

In case of the laminar flow you will have the velocity distribution like this but in case of the turbulent flow you will change the velocity distributions as we increase this flow Reynolds numbers. So to quantify the what could be the relationship for these type of velocity distribution $\frac{\bar{V}}{V_{\max}}$, it establish a relationship with $\left(\frac{y}{D/2}\right)^{1/n}$.

$$\frac{\bar{V}}{V_{\max}} = \left(\frac{y}{D/2}\right)^{1/n}$$

It is fitted with a data curve and finally find out the n varies with a Reynolds numbers. Its ranging from 6 to 10 for the Reynolds numbers from 4000 to the 3.2 million. So in this case and varies considering from the 6 to 10. So this n is a function of Reynolds numbers okay. But many of the case also n is considered is 7. So many of the time is called 1 by 7 for describing the velocity distributions in a pipe flow for turbulent regions.

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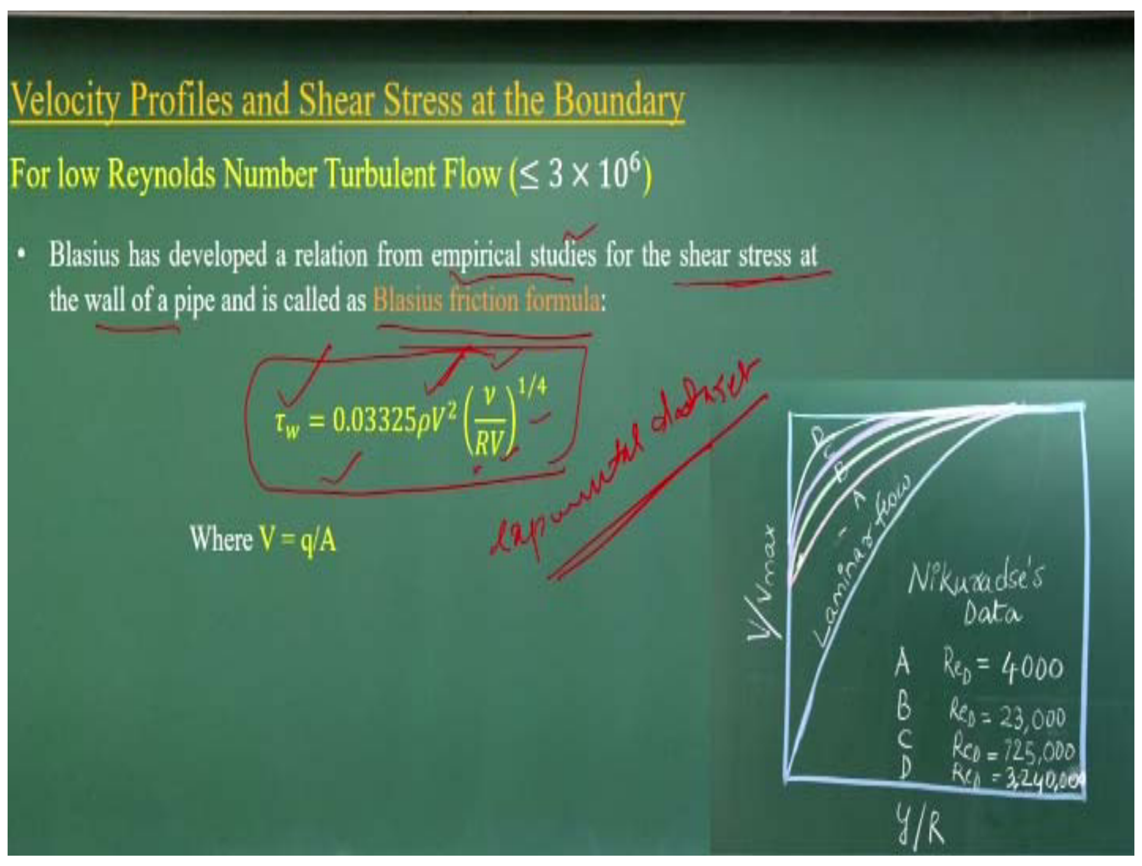

But when you look it what could be the shear stress that is what is the wall shear stress, shear stress at the wall of the pipe. That is what is also empirically established with the functions of wall stress, the average velocities and hydraulic radius and V. That is what is develop it to compute it what could be the wall stress from experimental data set, from experimental data set it derive.

$$\tau_w = 0.03325\rho V^2 \left(\frac{\nu}{RV}\right)^{1/4}$$

Where V = q/A

That is the reason is called empirical studies, the wall stress the relations with the average velocities and the kinematic viscosities and R value. That is giving the what could be the wall stress for a circular pipe.

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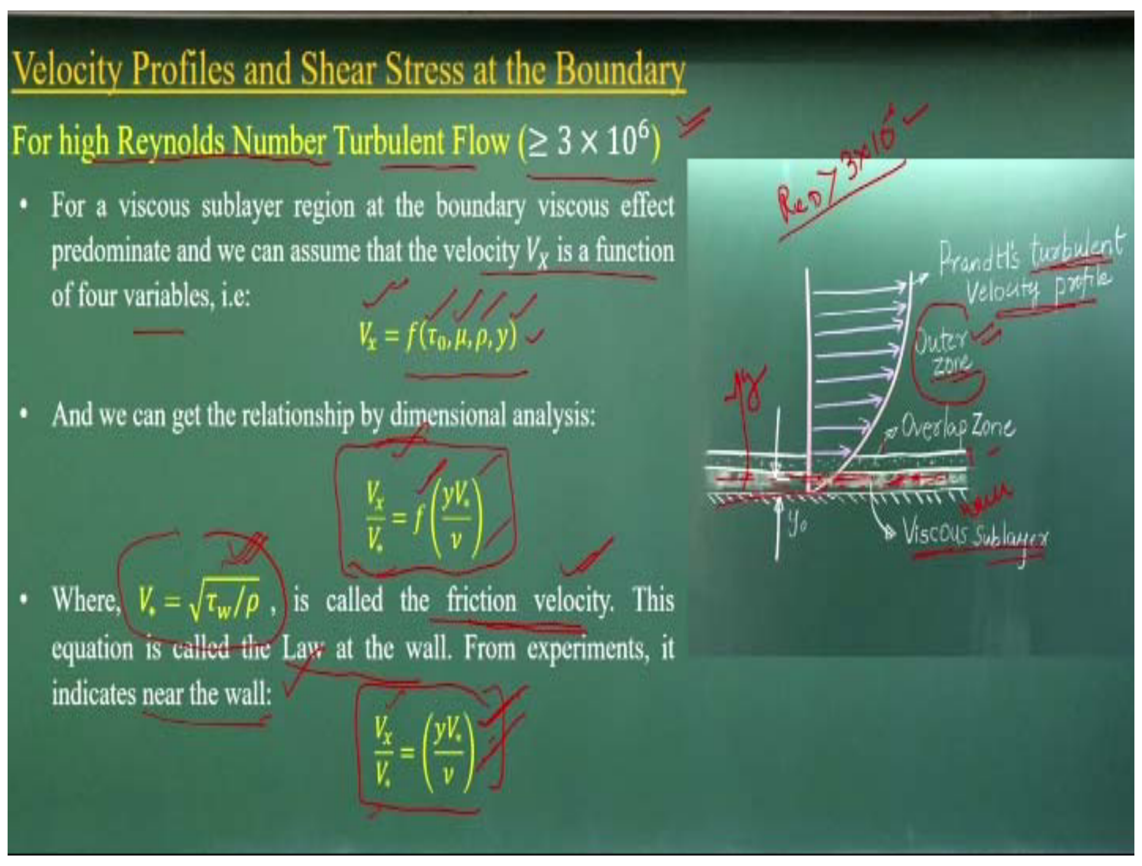

Now if you look it that when you go for higher Reynolds numbers turbulent flow, higher Reynolds number turbulent flow, which is more than 3 million, the Reynolds number is more than 3 millions. In this case what it actually happens is I am not going more details. When you have the Reynolds numbers more than 3 millions, then there is certain zone of the velocity profile develops.

Like at the wall you can see there will be a flow which behave like the laminar flow, which behave like viscous sublayers. And the you go outer of zones you will see that will be a velocity profile will be develop it, which is not the linear relationship between. In laminar flow there is a linear relationship between the shear stress and the velocity distribution. But that does not happen it when you go for the outer zones.

$$V_x = f(\tau_0, \mu, \rho, y)$$

And in between there is a overlap zones formations. In this lecture, we are not going more details, because I have not introduced you boundary layer concept, but try to understand it when you go for higher Reynolds numbers flow, you will have a very small thickness, near to the wall, where the flow behaves like a layer, a way the viscous is dominated too much and that is the reasons you will have a viscous sublayer zones.

But as go far away from the wall, you will have it where it will have a turbulent velocity profile zones, which experimentally established that change of reasons will happen as we will go from the wall. Most of the times this type of things as I said it earlier we do a non-dimensional analysis and then we try to establish the relationship with dependent variable or independent variable.

For this case if velocity distributions is considered is a functions of shear stress, wall shear stress, mu, rho and the y, y is the distance from the wall. And if you do a simple dimensional analysis we can compute like this. This is a simple dimensional analysis to rearrange the terms. You will get a friction velocity or the shear velocity in terms of wall stress. In terms of wall shear stress we will get it this value.

$$\frac{V_x}{V_*} = f\left(\frac{yV_*}{\nu}\right)$$

Where, $V_* = \sqrt{\tau_w/\rho}$

So this is the relationship. We do not know this f value and from the experiment we found that f value is just a linear functions okay, just a linear function. It does not have any, so we can find out the Vx, V_* is this way in the regions which is near to the wall okay. This is the regions near to the, the law at the wall which is very simple as a $\frac{V_x}{V_*}$. V_* is representing this friction velocity is a simple $\frac{yV_*}{\nu}$.

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